QUESTION

Two firms X and Y manufacture batteries with a nominal life of 1000 hrs. It is claimed that those manufactures by Y have a greater average life. 8 batteries are chosen at random from each manufacturer and their life-times (in hours above 1000) are:

X	34	-9	-20	11	-2	15	1	2
Y	40	-3	20	-2	16	18	10	13

Assuming that the battery life-times are normally distributed show that it is reasonable to assume that they come from populations with the same variance.

Test the claim of greater average life of Y batteries (a) using an appropriate t-test, (b) using ANOVA.

ANSWER

X	34	-9	-20	11	-2	15	1	2
Y	40	-3	20	-2	16	18	10	13

$H_0 : \sigma_1^2 = \sigma_2^2 \quad H_1 : \sigma_1^2 \neq \sigma_2^2 \quad \alpha = 5\%$

Test 6, comparison of two variances. Assume normal distribution $z = \frac{s_1^2}{s_2^2} \sim F_{n_1-1,n_2-1}$

$\overline{x} = 4$

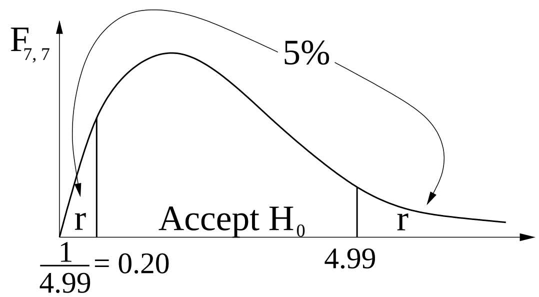
$s_1 = 16.3183, \quad n_1 = 8$

$\overline{y} = 14$

$s_2 = 13.5962, \quad n_2 = 8$

$z = \frac{(16.3183)^2}{(13.5962)^2} = 1.44$

not significant



Hence accept variances equal.

(a) $H_0 : \mu_1 = \mu_2 \quad H_1 < \mu_2$

Test 4, comparison of two means, variances unknown but equal.

$z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\{s^2(\frac{1}{n_1}+\frac{1}{n_2})\}}} \sim t_{n_1+n_3}$

$s^2 = \frac{7\times16.3183^2+7\times13.5962^2}{14}$

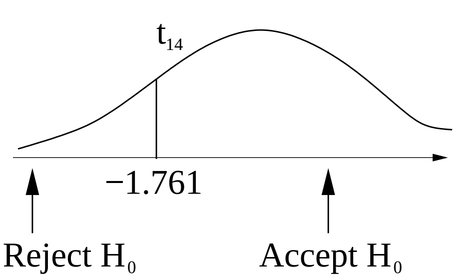
$s = 15.0190$

$z = \frac{4-14}{15.019\sqrt{\frac{1}{8}+\frac{1}{8}}}$

$= 1.33$

not significant

Hence accept H_0.



(b) $T_1 = 32$ $T_2 = 112$ $T = 144$ $C = \frac{(144)^2}{16} = 1296$

$\sum x^2$ $TSS = 4854 - 1296 = 3558$ $BSS = \frac{(32)^2}{8} + \frac{(112)^2}{8} - 1296 = 400$ $WSS = 3558 - 400 = 3158$. (Check that this is the numerator of $s^2 = \sum(x - \overline{x})^2 + \sum(y - \overline{y})^2$

Anova table

source	df	ss	ms
between groups	1	400	400
within groups	14	3158	225.57=σ^2
total	15*3558		

$H_0 : \mu_1 = \mu_2$ $H_1 \neq \mu_2$ $\alpha = 10\%$ will test $H_0 : \mu_1 = \mu_2$ against $H_1 < \mu_2$ at $\alpha = 5\%$

$F_{1,14} = \frac{400}{225,57} = 1.77 = (1.33)^2$ which is not significant.