QUESTION

If the point P has position vector $\mathbf{r} = \sin(2t)\mathbf{i} - \cos(2t)\mathbf{j} + t^3\mathbf{k}$ at time t, then find the speed of P as a function of t.

ANSWER

$$\begin{split} \mathbf{r} &= \sin(2t)\mathbf{i} - \cos(2t)\mathbf{j} + t^3\mathbf{k} \\ \dot{\mathbf{r}} &= 2\cos(2t)\mathbf{i} + 2\sin(2t)\mathbf{j} + 3t^2\mathbf{k} \\ \mathrm{Speed} &= |\dot{\mathbf{r}}| = \{4\cos^2(2t) + 4\sin^2(2t) + 9t^4\}^{\frac{1}{2}} = (4+9t^4)^{\frac{1}{2}} \end{split}$$