QUESTION

It is known from experience that the mean breaking strength of a particular brand of fibre is 9.25N with a standard deviation of 1.20N, where N denotes Newtons. A sample of 36 fibres was recently selected at random from the production and found to have a mean breaking strength of 8.90N.

(i) State the standard error of the sample mean.

(ii) Construct a 95% confidence interval for the mean breaking strength foe the fibre currently being produced, and determine how large a sample would have been required to obtain an interval with length less then 0.5N.

(iii) Using a 5% test of significance, can we conclude from the sample measurements that the strength of the fibre has changed?

ANSWER

We are given that $\mu = 9.25N,\ \sigma = 1.20N,\ n = 36$

(i) Standard error of sample mean is $\dfrac{\sigma}{\sqrt{n}} = \dfrac{1.20}{\sqrt{36}} = 0.20N$

(ii) The confidence interval is double sided, so the interval is

$\overline{x} \pm (1.96)(0.20) = 8.90 \pm 0.392$ i.e. 8.508 to 9.292

The interval length is $2(1.96)\dfrac{\sigma}{\sqrt{n}} = 2(1.96)\dfrac{(1.2)}{\sqrt{n}} = \dfrac{4.704}{\sqrt{n}}$

Length<0.5 if $\dfrac{4.704}{\sqrt{n}} < 0.5$, i.e. $\sqrt{n} > \dfrac{4.704}{0.5} = 9.408$ i.e. $n > (9.408)^2 = 88.5$ therefore sample size must be at least 89.

(iii) Hypothesis $H_0 : \mu = 9.25N,\ H_1 : \mu \neq 9.25N$ Standard deviation = 1.2N

This is a two sided hypothesis

Test procedure: accept H_0 if $\dfrac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} > -1.96$ or < 1.96 for a 5% significance test.

Now $\dfrac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{8.90 - 9.25}{0.20} = -\dfrac{0.35}{0.20} = -1.75$

This is inside the interval $(-1.96, 1.96)$, hence sample results suggest that the strength of the fibre has not changed.