Question

(i) Calculate the arc length of the curve γ in Qn.1(iv) from $(0,2)$ to $(2\pi,2)$.
(ii) Calculate the arc length of the curve γ in Qn.1(v) from $(1,0)$ to $(e^{2\pi k},0)$.
(iii) Calculate the path length of the curve $y^2=x^3$ from $(1,1)$ to $(-1,1)$.

Answer

(i)

$$\begin{aligned}
\gamma(t) &= (t-\sin t, 1+\cos t) \\
\|\gamma'(t)\| &= ((1-c)^2+s^2)^{\frac{1}{2}} \\
&= (2(1-c))^{\frac{1}{2}} \\
&= 2\sin\frac{t}{2}, \quad (\textbf{note } :\geq 0 \textbf{ for } 0 \geq y \geq 2\pi) \\
\Rightarrow \textbf{length} &= \int_0^{2\pi} 2\sin\frac{t}{2}\,dt \\
&= \left[-4\cos\frac{t}{2}\right]_0^{2\pi} \\
&= 4-(-4)=8, \quad (\textbf{Check } : 2\pi < 8 < 2\pi+4)
\end{aligned}$$

(ii)

$$\begin{aligned}
\gamma(t) &= (e^{kt}\cos t, e^{kt}\sin t) \\
\|\gamma'(t)\| &= e^{kt}((kc-s)^2+(ks+c)^2)^{\frac{1}{2}} \\
&= e^{kt}.(k^2+1)^{\frac{1}{2}} \\
\Rightarrow \textbf{length} &= \int_0^{2\pi} e^{kt}(k^2+1)^{\frac{1}{2}}\,dt \\
&= \frac{1}{k}(k^2+1)^{\frac{1}{2}}(e^{2\pi k}-1).
\end{aligned}$$

(iii)

$$\begin{aligned}
\gamma(t) &= (t^2,t^3), \ : \ -1 \geq t \geq 1 \\
\textbf{length} &= \int_{-1}^{1} \|(2t,3t^2)\|\,dt \\
&= \int_{-1}^{1} (4t^2+9t^4)^{\frac{1}{2}}\,dt \\
&= \int_{-1}^{1} |t|(4+9t^2)^{\frac{1}{2}}\,dt \\
&= 2\times\left[\frac{1}{27}(4+9t^2)^{\frac{3}{2}}\right]_0^1 \\
&= \frac{2}{27}(13^{\frac{3}{2}}-4^{\frac{3}{2}}).
\end{aligned}$$