

**Question**

Suppose that  $X$  has a uniform distribution on the interval  $(0,1)$ . Show that the pdf of  $Y = (8X)^{\frac{1}{3}}$  is given by

$$f(x) = \begin{cases} \frac{3}{8}y^2, & \text{for } 0 < y < 2; \\ 0, & \text{otherwise} \end{cases}$$

**Answer**

The transformation is

$$y = (8x)^{\frac{1}{3}} = 2x^{\frac{1}{3}}$$

Therefore the range of  $y$  is  $0 < y < 2$ .

Also  $x = \frac{y^3}{8}$ .

Therefore  $\frac{dx}{dy} = \frac{1}{8} \cdot 3 \cdot y^2$ .

Therefore the pdf of  $Y$  is

$$\begin{aligned} g(y) &= 1 \cdot \left| \frac{3}{8}y^2 \right|, \quad 0 < y < 2 \\ &= \frac{3}{8}y^2, \quad 0 < y < 2. \end{aligned}$$