Question
From the relations

$$\begin{aligned}
\mathbf{e}_r &= \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta \\
\mathbf{e}_\phi &= -\mathbf{i}\sin\phi + \mathbf{j}\cos\phi \\
\mathbf{e}_\theta &= \mathbf{i}\cos\theta\cos\phi + \mathbf{j}\cos\theta\sin\phi - \mathbf{k}\sin\theta
\end{aligned}$$

verify that $\mathbf{e}_r, \mathbf{e}_\phi, \mathbf{e}_\theta$, the basis vectors of spherical polar coordinates, form an orthogonal set, i.e. that they each have unit length and are mutually orthogonal.

Answer
Showing that the vectors have unit length:

$$\begin{aligned}
|\mathbf{e}_r| &= (\sin\theta\cos\phi)^2 + (\sin\theta\sin\phi)^2 + (\cos\theta)^2 \\
&= \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta \\
&= \sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta \\
&= \sin^2\theta + \cos^2\theta \\
&= 1
\end{aligned}$$

$$\begin{aligned}
|\mathbf{e}_\phi| &= (\sin\phi)^2 + (\cos\phi)^2 \\
&= 1
\end{aligned}$$

$$\begin{aligned}
|\mathbf{e}_\theta| &= (\cos\theta\cos\phi)^2 + (\cos\theta\sin\phi)^2 + (\sin\phi)^2 \\
&= \cos^2\theta\cos^2\phi + \cos^2\theta\sin^2\phi + \sin^2\phi \\
&= \cos^2\theta(\cos^2\phi + \sin^2\phi) + \sin^2\phi \\
&= \cos^2\theta + \sin^2\phi \\
&= 1
\end{aligned}$$

Showing they the vectors are mutually orthogonal:

$$\begin{aligned}
\mathbf{e}_r \cdot \mathbf{e}_\phi &= -\sin\theta\cos\phi\sin\phi + \sin\theta\cos\phi\sin\phi \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\mathbf{e}_r \cdot \mathbf{e}_\theta &= \cos\theta\cos^2\phi\sin\theta + \sin\theta\sin^2\phi\cos\theta - \cos\theta\sin\theta \\
&= \cos\theta\sin\theta(\sin^2\phi + \cos^2\phi) - \cos\theta\sin\theta \\
&= 0
\end{aligned}$$

$$\begin{aligned}\mathbf{e}_\phi \cdot \mathbf{e}_\theta &= -\cos\theta\cos\phi\sin\phi + \cos\phi\cos\phi\sin\phi \\ &= 0\end{aligned}$$