Question

The rate at which a radioactive substance splits up is given by

$$\frac{dN}{dt} = -\lambda N,$$

where N is the number of atoms present after t seconds and *lambda* is a constant. Show that $N = N_0 e^{-\lambda t}$ where N_0 is the number of atoms present initially. Find the times in *years* for half the number of atoms of a given mass of radium to disintegrate if $\lambda = 1.37 \times 10^{-11}$ per second.

Answer

$$\begin{aligned}
\frac{dN}{dt} &= -\lambda N \\
\Rightarrow \quad \int \frac{dN}{N} &= -\lambda \int dt \\
\Rightarrow \quad \ln N &= -\lambda t = c \\
\text{or} \quad N &= e^{-\lambda t + c} \\
N &= e^c e^{-\lambda t}
\end{aligned}$$

so let $e^c = N_0$

$\underline{N = N_0 e^{-\lambda t}}$

What is N_0?

Set $t = 0$ to get $N = N_0 e^{-\lambda \cdot 0} = N_0$ i.e., $N = N_0$ when $t = 0$.

We want the time, T, when $N = \dfrac{N_0}{2}$

i.e.,

$$\begin{aligned}
\frac{N_0}{2} &= N_0 e^{-\lambda T} \\
\Rightarrow \quad \frac{1}{2} &= e^{-\lambda T} \\
\ln\left(\frac{1}{2}\right) &= -\lambda T \\
\text{or} \quad T &= -\frac{1}{\lambda} \ln\left(\frac{1}{2}\right) \\
&= +\frac{\ln(2)}{1.37 \times 10^{-11} (per sec)} \\
&= 50,594,684,712 secs \\
&= \frac{50,594,684,712}{60 \times 60 \times 24 \times 365} years \\
&\underline{= 1604.3 \ years}
\end{aligned}$$