QUESTION

(a) Two products are obtained from the same supplier. For each product, the price of an item, the cost of placing an order, the holding cost per item per annum and the annual demand are shown in the following table. Demand for both products is steady, and shortages must not occur.

Product	Price(£)	Order cost (£)	Holding cost (£)	Demand
1	4	10	5	900
2	15	19	8	304

The supplier gives a discount of 2.5% whenever a buyer spends at least £500. Two types of ordering policy are possible: either order both products independently of each other, or always place an order for both products at the same time. Perform a cost analysis of the two policies, and hence recommend a policy to be adopted.

(b) In a batch production system, n products are manufactured on a single machine which can produce only one type of product at a time. The machine can produce any desired quantity of product after the necessary set-up procedure.

The following data are available for each product j $(j = 1, \dots, n)$.

Demand = d_j per month.

Production rate = r_j per month.

Stock holding cost = h_j per month.

Set-up time = t_j months.

Set-up cost = s_j.

Assume that demand is steady. A common cycle approach is to be adopted. Derive an expression for the monthly cost of this method, and hence find the optimal cycle length.

ANSWER

(a) For a single product, the annual cost is

$$K(Q) = \frac{sd}{Q} + \frac{1}{2}hQ + cd(1 - \text{discount})$$

Ignoring the discount, K is minimized when

$$\frac{dK}{dQ} = -\frac{sd}{Q^2} + \frac{1}{2}h = 0 \; Q* = \sqrt{\frac{2sd}{h}}$$

For product 1, $Q* = \sqrt{\frac{2.10.900}{5}} = 60$

$$K_1(60) = \frac{10.900}{60} + \frac{1}{2}.5.60 + 4.900 = 3900$$

To obtain discount $4Q_1 = 1200, Q_1 = 300$

$$K_1(300) = \frac{10.900}{300} + \frac{1}{2}5.300 + 4.900.0.975 = 4290$$

Thus, $Q_1 = 60$ is optimal order quantity for product 1. $K_1 = 3900$

For product 2, $Q_2* = \sqrt{\frac{2.19.304}{8}} = 38$

$$K_2(38) = \frac{19.304}{38} + \frac{1}{2}.8.38 + 15.304 = 4864$$

To obtain discount $15Q_2 = 1200 \; Q_2 = 80$

$$K_2(80) = \frac{19.304}{80} + \frac{1}{2}.8.80 + 15.304.0.975 = 4838.2$$

Thus $Q_2 = 80$ is optimal order quantity for product 2. $K_2 = 4838.2$

Suppose both products ordered together every time T time units.

$$K = \sum_{i=1}^{2} \left(\frac{S_i}{T} + \frac{1}{2}h_i d_i T \right)$$

Ignoring the discount, K is minimized when

$$\frac{dK}{dT} = -\frac{1}{T^2}\sum_{i=1}^{2} S_i + \frac{1}{2}\sum_{i=1}^{2} h_i d_i = 0 \; T* = \sqrt{\frac{2\sum S_i}{\sum h_i d_i}}$$

$T* = \sqrt{\frac{2(10+19)}{5.900+8.304}} = 0.09147$

To obtain the discount $Q_1C_1 + Q_2C_2 \geq 1200$

$$T(c_1d_1 + c_2d_2) \geq 1200 \; T \geq 0.14706$$

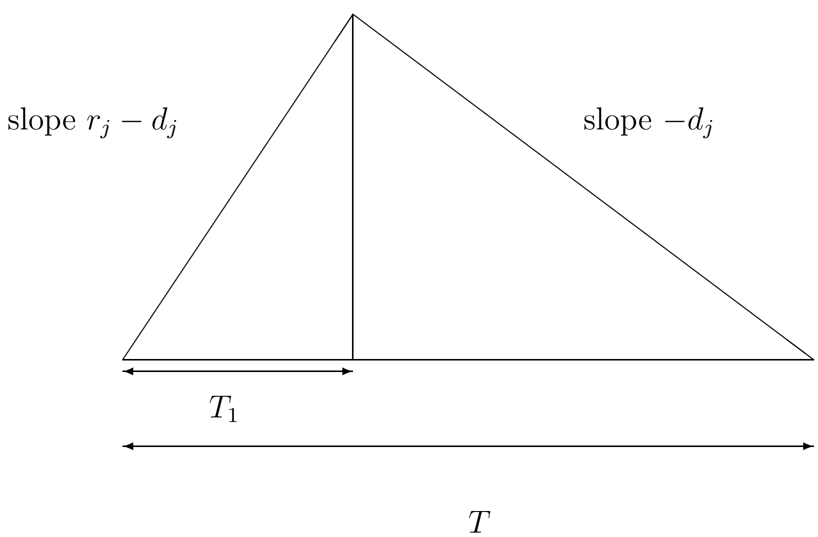
For $T = 0.14706$,

$$K = \frac{19}{0.14706} + \frac{1}{2}(5.900+8.304)0.14706 + (4.900+15.304)0.975 = 8594.91$$

For individual ordering, the total cost is 8738.2. Thus, order quantities 132.4 and 44.7 respectively at the same time.

(b) Let Q_j denote the production quantity of product j, and let T denote the cycle length.

Since $T_1 = \frac{Q_j}{r_j}$, the maximum stack level is $T_1(r_j - d_j) = Q_j\left(1 - \frac{d_j}{r_j}\right)$



Thus, the monthly cost is

$$K = \sum_{j=1}^{n}\left(\frac{S_j d_j}{Q_j} + \frac{1}{2}h_j Q_j\left(1 - \frac{d_j}{r_j}\right)\right)$$

For the common cycle method, $Q_j = d_j T$. Thus

$$K = \sum_{j=1}^{n}\left(\frac{S_j}{T} + \frac{1}{2}h_j Q_j\left(1 - \frac{d_j}{r_j}\right)\right)$$

$\frac{dK}{dT} = 0$ gives

$$-\sum_{j=1}^{n}\frac{s_j}{T^2} + \sum_{j=1}^{n}\frac{1}{2}h_j d_j\left(1 - \frac{d_j}{r_j}\right) = 0$$

$$T* = \sqrt{\frac{2\sum_{j=1}^{n} S_j}{\sum_{j=1}^{n} h_j d_j \left(1 - \frac{d_j}{r_j}\right)}}$$

There is a lower bound T^{LB} on T due to feasibility. In a unit time interval, set-ups and production must be done. Thus,

$$\sum_{j=1}^{n} \frac{t_j}{T} + \sum_{j=1}^{n} \frac{d_j}{r_j} \leq 1$$

to give

$$T^{LB} = \sum_{j=1}^{n} \frac{t_j}{\left(1 - \sum_{j=1}^{n} \frac{d_j}{r_j}\right)}$$

Choose $T = max\{T*, T^{LB}\}$