Question

Suppose that the pdf of a r.v. X is given by

$$f(x) = \begin{cases} c(9-x^2), & \text{for } -3 \le x \le 3, \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of the constant c. Find the cdf of X, and sketch the pdf and cdf of X. Find the values of the following probabilities:

$$P\{X < 0\},\ P\{-1 \le X \le 1\},\ P\{X > 2\}.$$

Answer

For $f(x)$ to be a pdf, it is necessary that

$$\int_{-\infty}^{\infty} f(u)\,du = \int_{-3}^{3} c(9-u^2)\,du = 1$$

and so $c = \frac{1}{36}$. Using the relationship between cdf and pdf

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(u)\,du \\ &= \int_{-3}^{x} \frac{9-u^2}{36}\,du \quad x \in (-3,3) \\ &= \frac{(18 + 9x - \frac{x^3}{3})}{36} \quad x \in (-3,3) \end{aligned}$$

Consequently,

$$\begin{aligned} &P\{X < 0\} = F(0) = \frac{1}{2}, \\ &P\{-1 \le X \le 1\} = F(1) - F(-1) = \frac{13}{27} \\ &P\{X > 2\} = 1 - F(2) = \frac{2}{27} \end{aligned}$$