**Question**

For each of the following functions, give a domain on which a continuous branch can be defined.

$(i)\mathrm{Log}(1+\mathrm{z})$, (ii) $\log(1+\mathrm{z})$, (iii)$\mathrm{Log}(1+\mathrm{z}^2)$, (iv)$(\mathrm{z}-1)^{\frac{1}{3}}$ (v)$(\mathrm{z}^2-1)^{\frac{1}{3}}$.

**Answer**

**(i)** $\underline{\mathrm{L}}\mathrm{og}(1+z)$ has $\mathrm{Arg}(1+z)$ i.e., $-\pi < \arg(1+z) \leq \pi$

so we need a branch cut from $z=-1$

PICTURE

**(ii)** $\log(1+z) = \log|1+z| + i\underbrace{\arg(1+z)}_{\text{what arg though?}}$

We need to define the branch of arg. Let's <u>choose</u> $0 < \arg(1+z) \leq 2pi$, then we have a branch point at $z=-1$ and a cut between $z=-1$ and $\underline{+\infty}$.

PICTURE

**(iii)** $\text{Log}(1+z^2) = \log|1+z^2| + i\text{Arg}(1+z^2)$

Branch points where $1 + z^2 = 0 \Rightarrow z = \pm i$

Now for Arg we have a cut where

PICTURE

**(iv)** $(z-1)^{\frac{1}{3}} = e^{\frac{1}{3}\log(z-1)} = e^{\frac{1}{3}\log|z-1| + \frac{i\arg(z-1)}{3}}$

But again, we need to choose a branch. If we choose Arg we have $-\pi < \arg(z-1) \leq \pi$ and we need a cut when $(z-1) < 0$, i.e.,

PICTURE

If we choose $0 < \arg(z) \leq 2pi$ we need a cut when $z - 1 > 0$, i.e.,

PICTURE

**(v)** $(z^2-1)^{\frac{1}{3}}$ has branch points where

$z^2 - 1 = 0 \Rightarrow z = \pm 1$

Again we have to choose the branch. If $\underline{\text{A}}\text{rg}$ we need cut where $(z^2-1) < 0$.

PICTURE

If we choose $0 < \arg(z^2-1) \leq 2\pi$ then we need a cut where $(z^2-1) > 0$

PICTURE

(Check this using the log-definitions.)