Question

Solve the initial value problem: $\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 1$

subject to $x(0) = \frac{dx(0)}{dt} = 0$

Answer

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 1$$

with $x(0) = \frac{dx(0)}{dt} = 0$

CF:

$$\begin{aligned} m^2 - m - 2 &= 0 \\ (m-2)(m+1) &= 0 \\ m &= 2, -1 \\ x_c &= Ae^{-t} + Be^{2t} \end{aligned}$$

Let $x^* = c$ substitute in $\frac{dx^*}{dt} = \frac{d^2x^*}{dt^2} = 0 \Rightarrow x^* = -\frac{1}{2}$

Thus $x(t) = Ae^{-t} + Be^{2t} - \frac{1}{2}$

Find A and B

$$\begin{aligned} x(0) &= A + B - \frac{1}{2} = 0 \\ \frac{dx(t)}{dt} &= -Ae^{-t} + 2Be^{2t} \\ \Rightarrow \frac{dx(0)}{dt} &= -A + 2B = 0 \\ \text{Hence} \quad A + B &= \frac{1}{2} \\ -A + 2B &= 0 \Rightarrow A = 2B,\ B = \frac{1}{6},\ A = \frac{1}{3} \end{aligned}$$

$$x(t) = \frac{1}{6}(2e^{-t} + e^{2t} - 3)$$