Question
Show that the triangle with vertices $\left(4, 0, 5\right)$, $\left(3, 6, 4\right)$ and $\left(1, 2, 3\right)$ has a right angle. (HINT: consider the edges of the triangle as vectors and calculate the angle between them.)

Answer
Let the triangle have vertices $A$, $B$ and $C$:

The vertices $A$, $B$ and $C$ have position vectors

$$A = \left< 4, 0, 5 \right> \quad B = \left< 3, 6, 4 \right> \quad C = \left< 1, 2, 3 \right>$$

respectively.

Find the vectors corresponding to the edges of the triangle:

$$\vec{AB} = \left< 3, 6, 4 \right> - \left< 4, 0, 5 \right> = \left< -1, 6, -1 \right> \quad \vec{BA} = -\vec{AB} = \left< 1, -6, 1 \right>$$

$$\vec{AC} = \left< 1, 2, 4 \right> - \left< 4, 0, 5 \right> = \left< -3, 2, -2 \right> \quad \vec{CA} = -\vec{AC} = \left< 3, -2, 2 \right>$$

$$\vec{BC} = \left< 1, 2, 3 \right> - \left< 3, 6, 4 \right> = \left< -2, -4, -1 \right> \quad \vec{CB} = -\vec{BC} = \left< 2, 4, 1 \right>$$

Remember: Two vectors $\vec{u}$ and $\vec{v}$ are orthogonal if and only if the dot product $\vec{u} \cdot \vec{v} = 0$.

Test $\theta_1$: $\vec{AB} \cdot \vec{AC} = \left\langle -1, 6, -1 \right\rangle \cdot \left\langle -3, 2, -2 \right\rangle = 3 + 12 + 2 = 17 \neq 0$ so angle $\theta_1$ is not a right angle.

Test $\theta_2$: $\vec{BA} \cdot \vec{BC} = \left\langle 1, -6, 1 \right\rangle \cdot \left\langle -2, -4, -1 \right\rangle = -2 + 24 - 1 = 21 \neq 0$ so angle $\theta_2$ is not a right angle.
Test $\theta_3$: $\overrightarrow{CA} \cdot \overrightarrow{CB} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} = 6 - 8 + 2 = 0$ so angle $\theta_3$ is a right angle as required, and hence the triangle does have a right angle.