Vector Functions and Curves
One variable functions

Question
A particle travels along the curve of intersection of the plane $x + y = 2$ and the cylinder $z = x^2$ in the direction of increasing y. The particle has constant speed $v = 3$, what is its velocity at the point $(1, 1, 1)$.
Answer
As the particle moves with increasing y on $x + y = 2$, $z = x^2$.
$\Rightarrow$ at time t

$$\underline{r} = (2 - g(t))\underline{i} + g(t)\underline{j} + (2 - g(t))^2\underline{k}$$

where $g(t)$ is an increasing function of time t.
$\Rightarrow$

$$\begin{aligned} \underline{v} &= \frac{dg}{dt}[-\underline{i} + \underline{j} - 2(2 - g(t))\underline{k}] \\ v &= \frac{dg}{dt}\sqrt{1 + 1 + 4(2 - g(t))^2} = 3 \end{aligned}$$

As the speed is 3.
When $g(t) = 1$, we have

$$\frac{dg}{dt} = 3\sqrt{6} = \sqrt{3/2}$$

$\Rightarrow$

$$\underline{v} = \sqrt{\frac{3}{2}}(-\underline{i} + \underline{j} - 2\underline{k}).$$