

Vector Functions and Curves
One variable functions

Question

Find the velocity, speed and acceleration of the particle with position given by $\underline{r}(t)$ at time t . Also determine the particles path.

$$\underline{r} = a \cos t \sin t \underline{i} + a \sin^2 t \underline{j} + a \cos t \underline{k}$$

Answer

Position:

$$\begin{aligned} \underline{r} &= a \cos t \sin t \underline{i} + a \sin^2 t \underline{j} + a \cos t \underline{k} \\ &= \frac{a}{2} \sin 2t \underline{i} + \frac{a}{2} (1 - \cos 2t) \underline{j} - a \sin t \underline{k} \end{aligned}$$

Velocity: $\underline{v} = a \cos 2t \underline{i} + a \sin 2t \underline{j} - a \sin t \underline{k}$

Speed: $v = a\sqrt{1 + \sin^2 t}$

Acceleration: $\underline{a} = -2a \sin 2t \underline{i} + 2a \cos 2t \underline{j} - a \cos t \underline{k}$

Path: the path lies on the sphere $x^2 + y^2 + z^2 = a^2$, on the surface defined in terms of spherical polar coordinates by $\phi = \theta$, on the circular cylinder $x^2 + y^2 = ay$, and on the parabolic cylinder $ay + z^2 = a^2$. Any two of these surfaces can be used to pin down the shape of the path.