

Vector Functions and Curves
One variable functions

Question

Find the velocity, speed and acceleration of the particle with position given by $\underline{r}(t)$ at time t . Also determine the particles path.

$$\underline{r} = at \cos \omega t \underline{i} + at \sin \omega t \underline{j} + b \ln 4t \underline{k}$$

Answer

Position: $\underline{r} = at \cos \omega t \underline{i} + at \sin \omega t \underline{j} + b \ln 4t \underline{k}$

Velocity: $\underline{v} = a(\cos \omega t - \omega t \sin \omega t) \underline{i} + a(\sin \omega t + \omega t \cos \omega t) \underline{j} + (b/t) \underline{k}$

Speed: $v = \sqrt{a^2(1 + \omega^2 t^2) + (b^2/t^2)}$

Acceleration:

$$\begin{aligned} \underline{a} &= -a\omega(2 \sin \omega t + \omega \cos \omega t) \underline{i} \\ &\quad + a\omega(2 \cos \omega t - \omega \sin \omega t) \underline{j} \\ &\quad - (b^2/t^2) \underline{k} \end{aligned}$$

Path: a spiral on the surface $x^2 + y^2 = a^2 e^{z/b}$.