**Vector Functions and Curves**
***One variable functions***

**Question**
An object travels on the curve given by the parameterization $\underline{r} = 3u\underline{i} + 3u^2\underline{j} + 2u^3\underline{k}$. Given that the object has a constant speed of 6 and that $u$ is increasing, find the velocity and acceleration of the object at the point $(3, 3, 2)$.
**Answer**

$$\begin{aligned}
\underline{r} &= 3u\underline{i} + 3u^2\underline{j} + 2u^3\underline{k} \\
\underline{v} &= \frac{du}{dt}(3\underline{i} + 6u\underline{j} + 6u^2\underline{k}) \\
\underline{a} &= \frac{d^2u}{dt^2}(3\underline{i} + 6u\underline{j} + 6u^2\underline{k}) \\
&\quad + \left(\frac{du}{dt}\right)^2 (6\underline{j} + 12u\underline{k})
\end{aligned}$$

As the speed of the object of 6 and $u$ is increasing,

$$\begin{aligned}
6 = |v| &= 3\frac{du}{dt}\sqrt{1 + 4u^2 + 4u^4} \\
&= 3(1 + 2u^2)\frac{du}{dt}
\end{aligned}$$

$\Rightarrow$

$$\frac{du}{dt} = \frac{2}{1 + 2u^2}, \text{ and}$$

$$\begin{aligned}
\frac{d^2u}{dt^2} &= \frac{-2}{(1 + 2u^2)^2} 4u \frac{du}{dt} \\
&= \frac{-16u}{(1 + 2u^2)^3}
\end{aligned}$$

It can be seen that the object is as $(3, 3, 2)$ when $u = 1$.
At this point $\frac{du}{dt} = 2/3$ and $\frac{d^2u}{dt^2} = -16/27$.
$\Rightarrow$

$$\begin{aligned}
\underline{v} &= \frac{2}{3}(3\underline{i} + 6u\underline{j} + 6u^2\underline{k}) \\
&= 2\underline{i} + 4\underline{j} + 4\underline{k} \\
\underline{a} &= -\frac{16}{27}(3\underline{i} + 6\underline{j} + 6\underline{k}) + \left(\frac{2}{3}\right)^2 (6\underline{j} + 12\underline{k}) \\
&= \frac{8}{9}(-2\underline{i} - \underline{j} + 2\underline{k})
\end{aligned}$$