Vector Functions and Curves
One variable functions

Question
An object travels on the curve of intersection of the cylinders $y = -x^2$ and $z = x^2$ with increasing x. When the particle is at $(1, -1, 1)$, it has a speed of $9cm/s$ which is increasing at a rate of $3cm/s^2$. Given that all distances are in cm, find the velocity and acceleration of the object at that point.
Answer

$$\begin{aligned}
\underline{r} &= x\underline{i} - x^2\underline{j} + x^2\underline{k} \\
\underline{v} &= \frac{dx}{dt}(\underline{i} - 2x\underline{j} + 2x\underline{k}) \\
\underline{a} &= \frac{d^2x}{dt^2}(\underline{i} - 2x\underline{j} + 2x\underline{k}) + \left(\frac{dx}{dt}\right)^2(-2\underline{j} + 2\underline{k}).
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
|\underline{v}| &= \left|\frac{dx}{dt}\right|\sqrt{1 + 4x^4 + 4x^4} \\
&= \sqrt{1 + 8x^4}\frac{dx}{dt},
\end{aligned}$$

as x is increasing.
At $(1, -1, 1)$, $x = 1$ and $|\underline{v}| = 9$, $\Rightarrow \frac{dx}{dt} = 3$.
$\Rightarrow$ at that point

$$\underline{v} = 3\underline{i} - 6\underline{j} + 6\underline{k}$$

.
Now

$$\frac{d}{dt}|\underline{v}| = \sqrt{1 + 8x^4}\frac{d^2}{dt^2} + \frac{16x^3}{\sqrt{1 + 8x^4}}\left(\frac{dx}{dt}\right)^2.$$

When $x = 1$, the left side is 3.
$\Rightarrow$

$$\begin{aligned}
3\left(\frac{d^2x}{dt^2}\right) + 48 &= 3 \\
\text{and} \quad \frac{d^2x}{dt^2} &= 15
\end{aligned}$$

at that point.
$\Rightarrow$ Acceleration at that point

$$\begin{aligned}
\underline{a} &= -15(\underline{i} - 2\underline{j} + 2\underline{k}) + 9(-2\underline{j} + 2\underline{k}) \\
&= -15\underline{i} + 12\underline{j} - 12\underline{k}
\end{aligned}$$