Vector Functions and Curves
Applications

Question
If run at full power, a self-propelled tank car can accelerate itself, when full (mass = Mkg) along a straight track at a m/sec^2.
At time $t = 0$ the tank is full. The contents leave the tank at a rate k kg/sec after that.
At time $t = 0$ the car is at rest and then full power is activated to accelerate the car forwards. How fast will it be moving at any time t before the tank is empty?
Answer
At time t seconds, the speed of the car is $v(t)$ and its mass is $m(t) = M - kt$kg.
When the car is at full power, since the motor accelerates the car at a m/s^2 the force applied is $F = ma$.
The force is the rate of change of momentum of the car (by Newton's Law).
$\Rightarrow$

$$\begin{aligned}
\frac{d}{dt}\left[(M-kt)v\right] &= Ma \\
(M-kt)\frac{dv}{dt} - kv &= Ma \\
\frac{dv}{Ma+kv} &= \frac{dt}{M-kt} \\
\frac{1}{k}\ln(Ma+kv) = -\frac{1}{k}\ln(M-kt) + \frac{1}{k}\ln C & \\
Ma + kv &= \frac{C}{M-kt}
\end{aligned}$$

When $t = 0$, $v = 0$, $\Rightarrow Ma = \frac{C}{M}$.
$\Rightarrow$

$$\begin{aligned}
C &= M^2a \\
kv &= \frac{M^2a}{M-kt} - Ma = \frac{Makt}{M-kt} \\
\Rightarrow v(t) &= \frac{Mat}{M-kt}\text{m/s}
\end{aligned}$$