QUESTION

(a) A project consists of activities $A, B, \ldots, L$ whose prerequisites are given in the table below. Draw a network, suitable for analysis by the critical path method, to represent the project. You should avoid using dummy activities, where possible. For each event, write the earliest and latest event times on the network, and deduce the critical path.

Activity	Prerequisites	Duration (days)
A	-	8
B	-	6
C	-	6
D	A	9
E	A,B	7
F	C	3
G	C	9
H	D	3
I	D,E,F	7
J	G	8
K	H,I	8
L	H,I	5

If the duration of activity E is increased to 10 days, discuss the effect on the overall project duration.

(b) An electronics company manufactures measuring instruments which have a reading device that must be correctly aligned. In some cases, the reading device gets out of alignment during delivery to customers. Such a fault occurs in 10% of deliveries. If the equipment is installed with an incorrect alignment, then the cost of correcting this fault is £5000.

The company has three possible courses of action. First, it can proceed with the installation of the equipment, and incurs no cost if the reading device is correctly aligned. Second, the company can send an engineer, at a cost of £400, to arrive just before installation. The engineer will check the alignment, and correct it, if necessary, prior to installation. Third, the company can make a field check of the equipment just before installation, at a cost of £100, with a view to deciding on whether to call an engineer before installation. However, the field check is not 100% reliable. Specifically, if the reading device is correctly aligned, the field check will indicate that adjustment is needed with probability 0.1; and if the reading device is not correctly aligned, the field check

will indicate that no adjustment is needed probability 0.2. To call an engineer after the field check, costs a further £500.

Develop a decision tree to find out whether the field check should be performed, and to decide in which situations an engineer should be called.

ANSWER
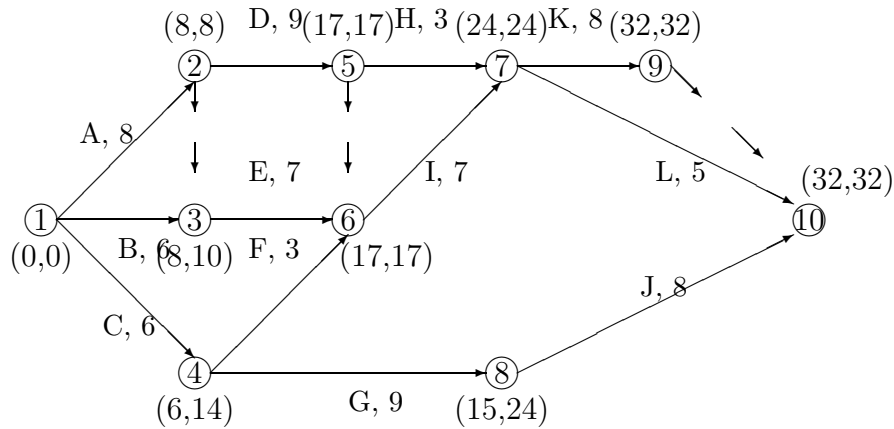
(a)

The critical path is 1 - 2 - 5 - 6 - 7 - 9 - 10, A - D - I - K

The total float for activity E is $TF_E = 17 - 7 - 8 = 2$

The increase in duration of 3 for E exceeds TF_E by 1 day, so the project duration becomes 33 days.

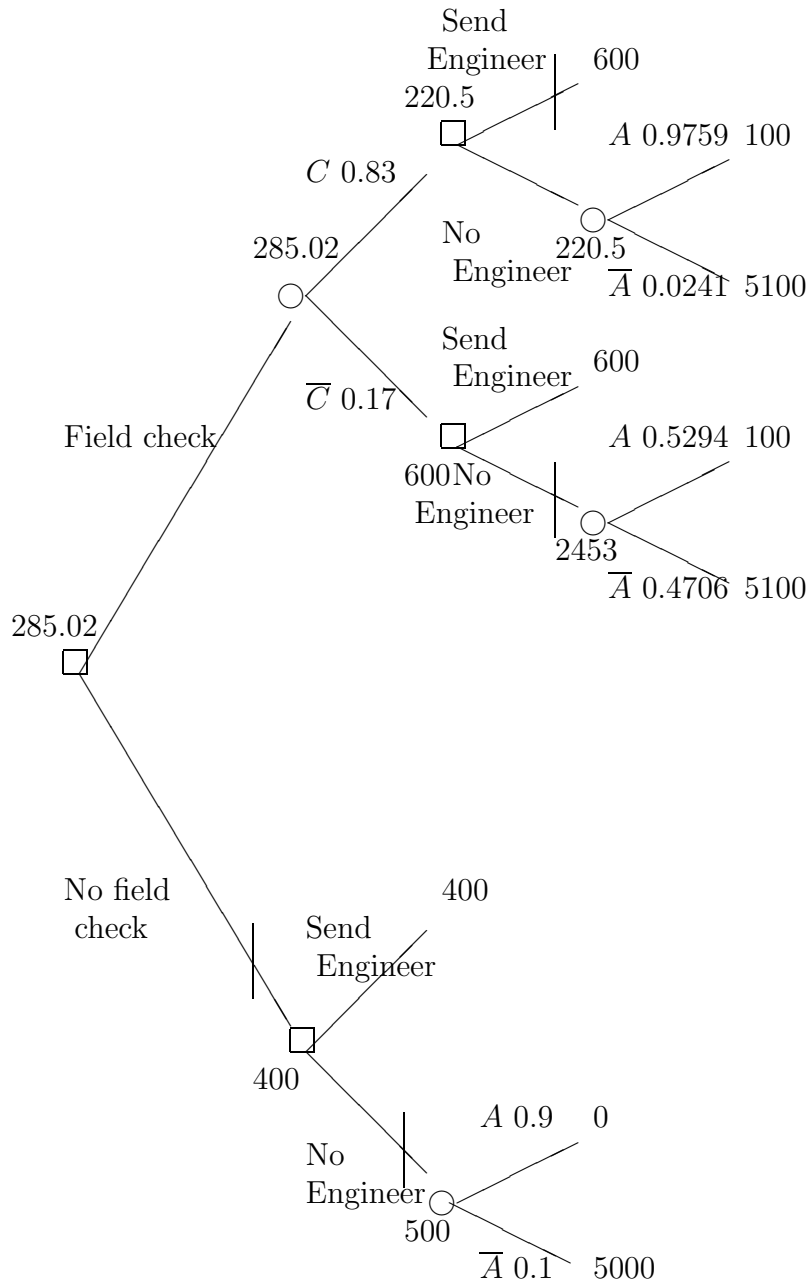**(b)** Let A be the event that the reading device is correctly aligned

$\overline{A}$ be the event that it is not

C be the event that the field test indicates correct alignment

$\overline{C}$ be the event that it does not.

$$
\begin{aligned}
P(A) &= 0.9 \\
P(\overline{A}) &= 0.1 \\
P(C|A) &= 0.9 \\
P(\overline{C}|A) &= 0.1 \\
P(C|\overline{A}) &= 0.2 \\
P(\overline{C}|\overline{A}) &= 0.8 \\
P(C) &= P(C|A)P(A) + P(C|\overline{A})P(\overline{A})
\end{aligned}
$$

$$
\begin{aligned}
&= 0.9 \times 0.9 + 0.2 \times 0.1 = 0.83 \\
P(\overline{C}) &= P(\overline{C}|A)P(A) + P(\overline{C}|\overline{A})P(\overline{A}) \\
&= 0.1 \times 0.9 + 0.8 \times 0.1 = 0.17 \\
P(A|C) &= \frac{P(C|A)P(A)}{P(C)} \\
&= \frac{0.9.9}{0.83} = 0.9759 \\
P(\overline{A}|C) &= \frac{P(C|\overline{A})P(\overline{A})}{P(C)} \\
&= \frac{0.2 \times 0.1}{0.83} = 0.0241 \\
P(A|\overline{C}) &= \frac{P(\overline{C}|A)P(A)}{P(\overline{C}} \\
&= \frac{0.1 \times 0.9}{0.17} = 0.5294
\end{aligned}
$$

Send
Engineer | 600
220.5

□

C 0.83

285.02

No
Engineer   A 0.9759 100
            220.5
○
            Ā 0.0241 5100

Field check

Send
Engineer   600

C̄ 0.17

□

600 No
Engineer   A 0.5294 100
        2453
        ○
        Ā 0.4706 5100

285.02

□

No field
check

400
Send
Engineer

□

400

No
Engineer   A 0.9   0

500 ○

Ā 0.1   5000

Thus, the field check should be undertaken. If it indicates that no

adjustment is necessary, then do not call the engineer; otherwise call the engineer.