QUESTION

(a) Solve the following linear programming problem using the simplex method.

$$
\begin{array}{ll}
\text{Maximize} & z = 28x_1 + 36x_2 + 4x_3 \\
\text{subject to} & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \\
& 4x_1 + 5x_2 + x_3 \leq 25 \\
& 2x_1 + 3x_2 + x_3 = 12 \\
& 6x_1 + 6x_2 - 2x_3 \geq 30.
\end{array}
$$

(b) A company produces hand-made rugs. The expected demand for rugs over the next six months is as follows.

Month	1	2	3	4	5	6
Demand	120	130	150	200	120	150

At the start of month 1, there are no rugs in stock. A worker takes 20 hours to make a rug. There are 20 workers, and each works for 140 hours per month, plus up to 40 hours per month of overtime. A worker is paid a fixed monthly wage for normal working, plus £30 per hour of overtime. At the end of each month, a holding cost of £10 is incurred for each rug in stock.

Write down a linear programming formulation (but do not attempt to solve it) for the problem of planning rug production and overtime working over the next six months so that demand is met at minimum total cost.

ANSWER

(a) Introduce slack variables $s_1 \geq 0, s_2 \geq 0$ and artificial variables $a_1, a_2 \geq 0$.

Basic	z'	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	
s_1	0	0	4	5	1	1	0	0	0	25
a_1	0	0	2	3	1	0	0	1	0	12
a_2	0	0	6	6	-2	0	-1	0	1	30
	1						1	1	0	
	1	0	-8	-9	1	0	1	0	0	-42
		1	-28	-36	-4	0	0	0	0	0

Basic	z'	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	
s_1	0	0	$\frac{2}{3}$	0	$-\frac{2}{3}$	1	0	$-\frac{5}{3}$	0	5
a_1	0	0	$\frac{2}{3}$	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	4
x_2	0	0	2	0	-4	0	-1	-2	1	6
	1	0	-2	0	4	0	1	3	1	-6
	0	1	-4	0	8	0	0	12	0	144

Basic	z'	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	
s_1	0	0	0	0	$\frac{2}{3}$	1	$\frac{1}{3}$	-1	$-\frac{1}{3}$	3
$x_1 0$	0	0	1	$\frac{5}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{3}$	2	
x_2	0	0	1	0	-2	0	$-\frac{1}{2}$	-1	$\frac{1}{2}$	3
	1	0	0	0	0	0	0	1	1	0
	0	1	0	0	0	0	-2	8	2	156

Basic	z	x_1	x_2	x_3	s_1	s_2	
s_1	0	0	-1	-1	1	0	1
x_1	0	0	3	5	0	1	6
s_2	0	1	$\frac{3}{2}$	$\frac{1}{2}$	0	0	6
	1	0	6	10	0	0	168

Solution is $x_1 = 6,\ x_2 = 0,\ x_3 = 0,\ z = 168$

(b) For each month j, let

x_j be the number of rugs produced

I_j be the end of month inventory

O_j be the number of hours overtime

Minimize $z = 10(I_1 + \ldots + I_6) + 30(O_1 + \ldots + O_6)$ subject to $x_j \geq 0, I_j \geq 0,\ O_j \geq 0\ j = 1, \ldots, 6$

$$\begin{aligned}
x_1 - I_1 &= 120 \\
I_1 + x_2 - I_2 &= 130 \\
I_2 + x_3 - I_3 &= 150 \\
I_3 + x_4 - I_4 &= 200 \\
I_4 + x_5 - I_5 &= 120 \\
I_5 + x_6 - I_6 &= 150 \\
20x_j &\leq 2800 + O_j\ j = 1, \ldots, 6 \\
O_j &\leq 800\ j = 1, \ldots, 6
\end{aligned}$$