Question Represent a perfect shuffle of a pack of 52 cards by the function $f:\ K \longrightarrow K$ where $K = \{0, 1, 2, 3, \cdots 51\}$ and f is given by

$$\left.\begin{array}{rcll} f(n) & = & 2n & (0 \leq n \leq 25) \\ & = & 2n - 51 & (26 \leq n \leq 51) \end{array}\right\}.$$

Show that every card returns to its original position after 8 shuffles. What would be the effect of introducing 2 jokers to the pack?

Answer Since $2^8 = 256 = 5 \times 51 + 1$ it follows that $f:\ K \longrightarrow K$ (which we can describe as $f(n) = 2n \bmod 51$) satisfies $f^8(n) \equiv n \bmod 51$ for every n. In fact there are two fixed points 0, 51 and a 2-cycle {17,34} which every other n has period 8 (i.e. six 8-cycles). With 54 cards we find that, apart from the fixed points 0, 53, every n has period 52: there is one 52-cycle.