QUESTION

Solve the following linear programming problem using the bounded variable simplex method.

$$\begin{array}{ll} \text{Maximize} & z = -13x_1 + 10x_2 + 8x_3 - 10x_4 - 22x_5 \\ \text{subject to} & -2x_1 + x_2 + 4x_3 - 2x_4 - 2x_5 \leq 5 \\ & -4x_1 + 2x_2 + x_3 - 2x_4 - 6x_5 \leq 13 \\ & 0 \leq x_1 \leq 1 \\ & 0 \leq x_2 \leq 20 \\ & 0 \leq x_3 \leq 4 \\ & 0 \leq x_4 \leq 3 \\ & 0 \leq x_5 \leq 5. \end{array}$$

(i) For the first constraint, give the range of values for the right-hand side within which the optimal basis remains unaltered. Also, perform this ranging analysis for the upper bound constraint $x_4 \leq 3$.

(ii) If the objective function coefficient of x_2 changes to $10+\delta$, for what range of values of δ is the change in the maximum value of z proportional to δ?

ANSWER

Add slack variables $s_1 \geq 0$, $s_2 \geq 0$ and use the bound variable simplex method.

Basic	z	x_1	x_2	x_3	x_4	x_5	s_1	s_2		Ratio
s_1	0	-2	1	4	-2	-2	1	0	5	5
s_2	0	-4	2	1	-2	-6	0	1	13	$\frac{13}{2}$
	1	3	-10	-8	10	22	0	0	0	

Basic	z'	x_1	x_2	x_3	x_4	x_5	s_1	s_2		Ratio
x_2	0	-2	1	4	-2	-2	1	0	5	$\frac{15}{2}$
s_2	0	0	0	-7	2	-2	-2	1	3	$\frac{3}{2}$
	1	-7	0	32	-10	2	10	0	50	

Basic	z'	x_1	x_2	x_3	x_4	x_5	s_1	s_2		Ratio
x_2	0	-2	1	-3	0	-4	-1	1	8	3
x_4	0	0	0	$-\frac{7}{2}$	1	-1	-1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
	1	-7	0	-3	0	-8	0	5	65	

Make substitution $x_4' = 3 - x_4$

Basic	z	x_1	x_2	x_3	x_4'	x_5'	s_1	s_2	
x_2	0	-2	1	11	4	0	3	-1	2+12=14
x_5	0	0	0	$\frac{7}{2}$	1	1	1	$-\frac{1}{2}$	$-\frac{3}{2}+3=\frac{3}{2}$
	1	-7	0	25	8	0	8	1	53+24=77

Make substitution $x_1' = 1 - x_1$

Basic	z	x_1'	x_2	x_3	x_4'	x_5	s_1	s_2	
x_2	0	2	1	11	4	0	3	-1	16
x_5	0	0	0	$\frac{7}{2}$	1	1	1	$-\frac{1}{2}$	$\frac{3}{2}$
	1	7	0	25	8	0	8	1	84

Optimal solution

$$x_1; = 0 \ x_2 = 16 \ x_3 = 0 \ x_4' = 0 \ x_5 = \frac{3}{2}$$

$$x_1 = 1 \ x_2 = 16 \ x_3 = 0 \ x_4 = 3 \ x_5 = \frac{3}{2} \ z = 84$$

(i) If the right-hand side of the first constraint is $5 + \delta$, the right hand sides in the final tableau are $16 + 3\delta \ \frac{3}{2} + \delta$.

For non-negativity $16 + 3\delta \geq 0, \ \delta \geq -\frac{16}{3}, \ \frac{3}{2} + \delta \geq 0 \ \delta \geq -\frac{3}{2}$

For variables to remain within bounds $16 + 3\delta \leq 20 \ \delta \leq \frac{4}{3}, \ \frac{3}{2} + \delta \leq 5 \ \delta \leq \frac{7}{2}$

Therefore the required range is $-\frac{3}{2} \leq \delta \leq \frac{4}{3}$

If the upper bound constraint is $x_4 \leq 3 + \delta$, then the right-hand sides in the final tableau become $16 + 4\delta, \ \frac{3}{2} + \delta$.

For non negativity, $16 + 4\delta \geq 0 \ \delta \geq -4, \ \frac{3}{2} + \delta \geq 0 \ \delta \geq -\frac{3}{2}$

For variables to remain within their lower bounds $16 + 4\delta \leq 20 \ \delta \leq 1, \ \frac{3}{2} + \delta \leq 5 \ \delta \leq \frac{7}{2}$

Therefore the required range is $-\frac{3}{2} \leq \delta \leq 1$

(ii) The z row of the final tableau becomes

$$7 + 2\delta \quad 0 \quad 25 + 11\delta \quad 8 + 4\delta \quad 0 \quad 8 + 3\delta \quad 1 - \delta \quad 84 + 16\delta$$

For non-negativity $-2 \leq \delta \leq 1$