QUESTION
State Burnside's Formula, carefully defining the terms used in the formula. Describe the elements of the rotation group of the cube, giving the order of each element, its fixed set, and describing the orbits of the faces for each rotation.
Use this to find the number of distinct ways there are to label the faces of a cube with five colours, where each colour may be used more than once. (As usual, "distinct" means that the labellings can be distinguished up to a rotation of the cube, so you will need to consider the action of the rotation group of the cube on the set of all possible labellings.)
ANSWER
Burnsides formual:

$$r|G| = \sum_{g \in G} |X_g|$$

where G acts on a set X, $X_g = \{x \in X | gx = x\}$ and $r=$ number of orbits.
Description

type	transformation	number	order	face orbits
A	identity	1	1	6
B	rotations of order 3 about diagonal	8	3	2
C	rotations of order 2 about line bisecting 2 opposite edges	6	2	3
D	rotations about lines joining midpoints of opposite faces	3	2	4
		6	4	3

Number of distinguishable dice=number of orbits of rotation group on all colourings.
By Burnside's formula

$$\begin{aligned} 24r &= 5^6 + 8.5^2 + 6.5^3 + 3.5^4 + 6.5^3 \\ &= 19200 \end{aligned}$$

so $r = \frac{19200}{24} = 800$.