Exam Question
Topic: Laplace
The function f is defined in terms of the function g by

$$f(x) = \int_0^x g(t)dt.$$

Write down the definition of the Laplace transform of $f(x)$. this gives a repeated integral. Reverse the order of integration and evaluate the inner integral.
Deduce that $L(g) = pL(f)$, where L denotes the Laplace transform.

Solution

$$\begin{aligned} L(f(x) &= \int_0^\infty \mathrm{e}^{-px} \int_0^x g(t)\,dt = \int_0^\infty dt \int_t^\infty \mathrm{e}^{-px} g(t)\,dt \\ &= \frac{1}{p}\int_0^\infty \mathrm{e}^{-pt} g(t)\,dt = \frac{1}{p}L(g). \\ \text{So } L(g) &= pL(f). \end{aligned}$$