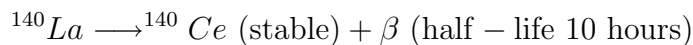
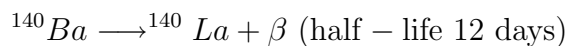


Question

^{140}Ba decays to give ^{140}La , which subsequently decays to ^{140}Ce , i.e.



Calculate the decay constants λ_{Ba} and λ_{La} . If the initial concentration of ^{140}Ba is 1000 units and the initial concentration of ^{140}La is zero, calculate the concentration of ^{140}Ba after 20 days and the concentrations of ^{140}La after both 10 hours and 20 days.

Note:

- (i) Pay particular attention to units;
- (ii) If the number of an isolated radioactive element is given by $N(t) = N(0)e^{-\lambda t}$, then λ is the decay constant;
- (iii) The results of Question 2 may be used.

Answer

Assume that time t is measured in hours. Let $B(t)$, $L(t)$ be the number of atoms of ^{140}Ba , ^{140}La at time t respectively.

The decay from ^{140}Ba to ^{140}La is modelled by the differential equation

$$\frac{\partial B}{\partial t} = -\lambda_{\text{Ba}}B$$

which gives $B(t) = B(0)e^{-\lambda_{\text{Ba}}t}$

Number of ^{140}Ba atoms after 12 days (288 hours) = $B(288) = B(0)e^{-\lambda_{\text{Ba}}288}$

Since half life of ^{140}Ba is 288 hours,

$$\frac{B(288)}{B(0)} = e^{-\lambda_{\text{Ba}}288} = \frac{1}{2}$$

so

$$\lambda_{\text{Ba}} = -\frac{\ln\left(\frac{1}{2}\right)}{288}L^{-1} \approx 2.407 \times 10^{-3}L^{-1}$$

Similarly, modelling decay from ^{140}La to ^{140}Ce by the differential equation

$$\frac{\partial L}{\partial t} = -\lambda_{\text{La}}L$$

gives

$$\lambda_{La} = -\frac{\ln\left(\frac{1}{2}\right)}{10}L^{-1} \approx 6.931 \times 10^{-2}L^{-1}$$

The decay processes are modelled by the equations found in Question 2, where

$$\begin{aligned}\alpha &= B(0) = 1000 \\ K_1 &= \lambda_{Ba} \\ K_2 &= \lambda_{La}\end{aligned}$$

This gives

$$B(t) = B(0)e^{-\lambda_{Ba}t} = 1000e^{-2.407 \times 10^{-3} \times t}$$

$$\begin{aligned}L(t) &= \frac{\lambda_{Ba}B(0)}{\lambda_{La} - \lambda_{Ba}} \left(e^{-\lambda_{Ba}t} - e^{-\lambda_{La}t} \right) \\ &\approx 35.977 \left(e^{-2.407 \times 10^{-3} \times t} - e^{-6.931 \times 10^{-2} \times t} \right)\end{aligned}$$

$$\begin{aligned}\text{Concentration of } ^{140}\text{Ba after 20 days (480 hours)} &= B(480) \approx 315\text{units} \\ \text{Concentration of } ^{140}\text{La after 10 hours} &= L(10) \approx 17\text{units} \\ \text{Concentration of } ^{140}\text{La after 20 days (480 hours)} &= L(480) \approx 11\text{units}\end{aligned}$$