Question

^{140}Ba decays to give ^{140}La, which subsequently decays to ^{140}Ce, i.e.

$$^{140}Ba \longrightarrow ^{140}\ La + \beta \text{ (half} - \text{life 12 days)}$$

$$^{140}La \longrightarrow ^{140}\ Ce \text{ (stable)} + \beta \text{ (half} - \text{life 10 hours)}$$

Calculate the decay constants λ_{Ba} and λ_{La}. If the initial concentration of ^{140}Ba is 1000 units and the initial concentration of ^{140}La is zero, calculate the concentration of ^{140}Ba after 20 days and the concentrations of ^{140}La after both 10 hours and 20 days.

Note:

(i) Pay particular attention to units;

(ii) If the number of an isolated radioactive element is given by $N(t) = N(0)e^{-\lambda t}$, then λ is the decay constant;

(iii) The results of Question 2 may be used.

Answer

Assume that time t is measured in hours. Let $B(t),\ L(t)$ be the number of atoms of ^{140}Ba,$^{140}\ La$ at time t respectively.

The decay from ^{140}Ba to ^{140}La is modelled by the differential equation

$$\frac{\partial B}{\partial t} = -\lambda_{Ba} B$$

which gives $B(t) = B(0)e^{-\lambda_{Ba} t}$

Number of ^{140}Ba atoms after 12 days (288 hours) $= B(288) = B(0)e^{-\lambda_{Ba} 288}$

Since half life of ^{140}Ba is 288 hours,

$$\frac{B(288)}{B(0)} = e^{-\lambda_{Ba} 288} = \frac{1}{2}$$

so

$$\lambda_{Ba} = -\frac{\ln\left(\frac{1}{2}\right)}{288} L^{-1} \approx 2.407 \times 10^{-3} L^{-1}$$

Similarly, modelling decay from ^{140}La to ^{140}Ce by the differential equation

$$\frac{\partial L}{\partial t} = -\lambda_{La} L$$

gives

$$\lambda_{La} = -\frac{\ln\left(\frac{1}{2}\right)}{10} L^{-1} \approx 6.931 \times 10^{-2} L^{-1}$$

The decay processes are modelled by the equations found in Question 2, where

$$\begin{aligned} \alpha &= B(0) = 1000 \\ K_1 &= \lambda_{Ba} \\ K_2 &= \lambda_{La} \end{aligned}$$

This gives

$$B(t) = B(0)e^{-\lambda_{Ba}t} = 1000e^{-2.407\times10^{-3}\times t}$$

$$\begin{aligned} L(t) &= \frac{\lambda_{Ba}B(0)}{\lambda_{La} - \lambda_{Ba}} \left(e^{-\lambda_{Ba}t} - e^{-\lambda_{La}t}\right) \\ &\approx 35.977 \left(e^{-2.407\times10^{-3}\times t} - e^{-6.931\times10^{-2}\times t}\right) \end{aligned}$$

Concentration of ^{140}Ba after 20 days (480 hours)	=	$B(480)$	$\approx$	$315 units$
Concentration of ^{140}La after 10 hours	=	$L(10)$	$\approx$	$17 units$
Concentration of ^{140}La after 20 days (480 hours)	=	$L(480)$	$\approx$	$11 units$