Question
Verify the chain rule

$$D(F\circ G)(p) = DF(q).DG(p),\ \ q = G(p)$$

where F, G **are the maps in question 1(ii), (i) respectively. Do likewise for the maps in (iii),(ii).**
Answer
With F**,** G **as in (ii), (i) respectively we have**

$$F\circ G(x_1,x_2) = (x_2^2 + 2x_2 + 4\sin x_1x_2 + 3(x_1-x_2)^2, x_2^2 + 8x_2 + 10\sin x_1x_2 + 6(x_1-x_2)^2)$$

so

$$D(F\circ G)(x_1,x_2) =$$

$$\begin{pmatrix} 4x_2\cos x_1x_2 + 6(x_1-x_2) & 2x_2 + 2 + 4x_1\cos x_1x_2 - 6(x_1-x_2) \\ 10x_2\cos x_1x_2 + 12(x_1-x_2) & 8x_2 + 8 + 10x_1\cos x_1x_2 - 12(x_1-x_2) \end{pmatrix}$$

which is the same as the product of matrices in (ii), (i).
Likewise for F**,** G **as in (iii), (ii) respectively we have**

$$\begin{aligned} F\circ G(x_1,x_2,x_3) &= 2(x_1+2x_2+3x_3)^2 + (x_1+2x_2+3x_3)(4x_1+5x_2+6x_3) \\ &\quad -(4x_1+5x_2+6x_3)^2 \\ &= -10x_1^2 - 7x_2^2 - 19x_1x_2 - 9x_2x_3 - 18x_1x_3 \end{aligned}$$

So

$$\begin{aligned} D(F\circ G)(x_1,x_2,x_3) &= d(F\circ G)(x_1,x_2,x_3) \\ &= (-20x_1 - 19x_2 - 18x_3, -14x_2 - 19x_1 - 9x_3, -9x_2 - 18x_1) \\ &= DF(y_1,y_2).DG(x_1,x_2,x_3) \end{aligned}$$

where $(y_1,y_2) = G(x_1,x_2,x_3)$**.**
as

$$\begin{aligned} DF(y_1,y_2) &= (4y_1 + y_2, y_1 - 2y_2) \\ &= (4(x_1+2x_2+3x_3) + (4x_1+5x_2+6x_3), \\ &\quad (x_1+2x_2+3x_3) - 2(4x_1+5x_2+6x_3)) \\ &= (8x_1 + 13x_2 + 18x_3, -7x_1 - 8x_2 - 9x_3) \end{aligned}$$

and DG **as in (ii).**