Question
Write down the Jacobian matrix $DF(p)$ for $F : \mathbf{R}^n \to \mathbf{R}^m$ at a typical point $p \in \mathbf{R}^n$:

(i) $F : \mathbf{R}^2 \to \mathbf{R}^3$ $F(x_1, x_2) = (x_2^2 + 2x_2, 2\sin x_1x_2, (x_1 - x_2)^2)$

(ii) $F : \mathbf{R}^3 \to \mathbf{R}^2$ $F(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, 4x_1 + 5x_2 + 6x_3)$

(iii) $F : \mathbf{R}^2 \to \mathbf{R}$ $F(x_1, x_2) = 2x_1^2 + x_1x_2 - x_2^2$.

Answer

(i)

$$DF(p) = \begin{pmatrix} 0 & 2x_2 + 2 \\ 2x_2 \cos x_1x_2 & 2x_1 \cos x_1x_2 \\ 2(x_1 - x_2) & -2(x_1 - x_2) \end{pmatrix}, \quad p = (x_1, x_2).$$

(ii)

$$DF(p) = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad p = (x_1, x_2, x_3).$$

[Here F is linear and is therefore its own derivative: the same at every point p.]

(iii)

$$DF(p) = (4x_1 + x_2, x_1 - 2x_2) \quad (= dF(p)), \quad p = (x_1, x_2).$$