QUESTION

Using l'Hopital's rule, or otherwise, evaluate $\lim\limits_{x \to 0} \left(\dfrac{x \sinh x}{\cosh x - 1} \right)$.

ANSWER

As $x \to 0$ then $\left(\dfrac{x \sinh x}{\cosh x - 1} \right) \to \dfrac{0}{0}$, an indeterminate form so using l'Hopital's rule gives

$\left(\dfrac{x \cosh x + \sinh x}{\sinh x} \right) \to \dfrac{0}{0}$, also an indeterminate form.

Using l'Hopital's rule again gives

$\lim\limits_{x \to 0} \left(\dfrac{x \sinh x + \cosh x + \cosh x}{\cosh x} \right) = \dfrac{0 + 1 + 1}{1} = 2$