

QUESTION

Find a particular integral of the differential equation $\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = e^{2t}$.

ANSWER

For the particular integral try $x = Ce^{2t}$, $\frac{dx}{dt} = 2Ce^{2t}$, $\frac{d^2x}{dt^2} = 4Ce^{2t}$

Substituting this into the differential equation gives

$$4Ce^{2t} + 2Ce^{2t} + Ce^{2t} = e^{2t}, \quad 7C = 1, \quad C = \frac{1}{7}$$

Therefore the particular integral is $x = \frac{1}{7}e^{2t}$.