QUESTION

You wish to leave a fund which pays out a fixed amount $p$ each year in perpetuity. If the interest rate is assumed to be $r$ from now until the end of time, how much should the initial fund be? If you wish to build up that fund by contributions over $n$ years, how much should your annual payment be.

ANSWER

Want an amount $P$ paid out after $n$ years for all time. Interest rate $= r$, assume annual compounding. Consider 2 sides: payments in and payments out.

From mortgage calculation above over $n$ years with an annual payment of $d$ you save a total of $\frac{d[(1+r)^n-1]}{r}$ (C)

If you want to pay out a sum $P$ for ever you need the following amounts saved, assuming payment at the end of the year:

| Year 1 | Year 2 | Year 3 | ... | Year n |
|---|---|---|---|---|
| $\frac{P}{(1+r)}$ | $\frac{P}{(1+r)^2}$ | $\frac{P}{(1+r)^3}$ | | $\frac{P}{(1+r)^n}$ |
| amount needed at start of year to pay $P$ in year 1 | amount needed at start of year to pay $P$ in year 2 | amount needed at start of year to pay $P$ in year 3 | | amount needed at start of year to pay $P$ in year $n$ |

The total required to pay $P$ at the end of each year in perpetuity is:

$$\sum_{i=1}^{\infty} \frac{P}{(1+r)^i} = P\left[\frac{1}{1-\frac{1}{(1+r)}}\right] - P = \frac{P}{r} \ (D)$$

Thus (C) must equal (D). Hence

$$\frac{P}{r} = \frac{d[(1+r)^n-1]}{r} \Rightarrow P = d[(1+r)^n-1] \Rightarrow d = \frac{P}{[(1+r)^n-1]}$$

Hence if you want to bequeath a prize of £1000 per year in 35 years time and expect to get 5% p.a. interest,

$$d = \frac{1000}{((1.05)^{35}-1)} = 221.43 \text{ per year.}$$

(Note that with inflation the 1000 would be worth increasingly less!)