QUESTION

A mortgage of £250,000 is to be paid of in equal monthly installments over 25 years. The interest rate is assumed to be constant at 8%. What are the monthly repayments?

ANSWER

This is a different calculation from above. Let mortgage be M, number of years to be paid off be n, interest rate be r, annual payment be p. Values of payment in each year:

	Year 1	Year 2	...	year j	...	Year n
Pay P in year 1$\rightarrow$	P	$P(1+r)$		$P(1+r)^{j-1}$		$P(1+r)^{n-1}$
Pay P in year 2$\rightarrow$	-	P		$P(1+r)^{j-2}$		$P(1+r)^{n-2}$
$\vdots$		$\ddots$		$\ddots$		$\vdots$
Pay P in year $n-1\rightarrow$						$P(1+r)$
pay$P$ in year $n\rightarrow$						P

So the total value of payments is the sum of the last column.

$$=P\sum_{i=0}^{n-1}(1+r)^i = P\frac{[(1+r)^n-1]}{(1+r-1)} = \frac{P(1+r)^n-1}{r}\ (A)$$

However, in this time the mortgage debt has increased to $m(1+r)^n$ (B)

Thus to pay of the mortgage we need (A)=(B)

$$\Rightarrow \left(\frac{(1+r)^n-1}{r}\right) = m(1+r)^n$$

$$\Rightarrow P = \frac{M(1+r)^n r}{(1+r)^n-1}$$

You may say "Ah, but you pay off P of the outstanding amount each year, so (B) isn't as much as this." Repeat this calculation on the basis of the "amount left to pay" after each year and you will see that the final answer is the same!

Thus if $M=250,000,\ r=0.08,\ n=25$

$$P=\frac{250,000\times(1.08)^{25}\times 0.08}{(1.08)^25-1} = 23,419.69$$

or

$$\frac{p}{12} = 1951,64$$

per month.

Note that mortgage companies normally charge annual compounding and credit payments at the end of the year (although perhaps charge interest from the beginning.) (NB average house price in UK at 1999 £65K, although this is clearly not realistic in London.)

$$\Rightarrow \frac{p}{12} = \begin{cases} 507.43@8\% \\ 464.81@7\% \\ 423.73@6\% \end{cases}$$

Here see how much small changes in interest rates can affect your monthly payments!