**Question**

Prove or give a counterexample to the following statement: if

$$\sum_{n=1}^{\infty} a_n$$

is a convergent infinite series of positive terms, then the power series

$$\sum_{n=1}^{\infty} a_n x^n$$

converges for all real numbers $x$.

**Answer**

The statement is false: to take a specific example, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, but the power series $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$ has radius of convergence 1, for instance by the ratio test.