**Question**

Use the Mean Value theorem to prove that if $f$ and $g$ are two differentiable functions on the closed interval $[a, b]$, where $a < b$, and if $f'(x) = g'(x)$ for all $x$ in $[a, b]$, then there is a constant $K$ so that $f(x) = g(x) + K$ for all $x$ in $[a, b]$.

**Answer**

Set $h(x) = f(x) - g(x)$, so that $h'(x) = f'(x) - g'(x) = 0$ for all $x$. Take $x$ in $(a, b]$, and apply the mean value theorem to $h(x)$ (which is continuous on $a, b$ and differentiable on $(a, b)$ since both $f(x)$ and $g(x)$ are) on $[a, x]$, to see that there exists $c$ in $(a, x)$ so that $h'(c) = \frac{h(x)-h(a)}{x-a}$. But since $h'(c) = 0$, we have that $h(x) - h(a) = 0$, or that $h(x) = h(a)$. That is, $f(x) = g(x) + h(a)$, as desired, where $K = h(a)$.