

Question

Determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

converges absolutely, converges conditionally, or diverges. (You do not need to evaluate the sum of the series in the case that it converges.)

Answer

Notice that this is an alternating series. Since $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$ and since $\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$, the alternating series test yields that this series converges.

However, the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges, for instance by comparison to the harmonic series, as $\frac{1}{\sqrt{n}} \geq \frac{1}{n}$ for all $n \geq 1$, and so this series does not converge absolutely. That is, this series converges conditionally.