Question

Prove that if a sequence $\{a_n\}$ is increasing and bounded above, then it is convergent.

Answer

Since $\{a_n\}$ is bounded above, it has a supremum a. By the definition of supremum, for every $\varepsilon > 0$, there exists M so that $|a_M - a| < \varepsilon$. Since $\{a_n\}$ is increasing and since a is an upper bound for $\{a_n\}$, we have that $a_M < a_n \leq a$ for every $n > M$. In particular, we have that $|a_n - a| < |a_M - a| < \varepsilon$ for every $n > M$, and this is just the definition that $\{a_n\}$ converges to a.