Question
Define what it means for a function $f : \mathbf{R} \to \mathbf{R}$ to be continuous. Using the definition, show that the function $f(x) = 2x - 5$ is continuous.

Answer
f is continuous at a if $\lim_{x \to a} f(x) = f(a)$. f is continuous if it is continuous at every point in its domain.

To show that $f(x) = 2x - 5$ is continuous, we show that it is continuous at a for every a. That is, we need to show that

$$\lim_{x \to a}(2x - 5) = 2a - 5.$$

So, for any $\varepsilon > 0$, take $\delta = \frac{1}{2}\varepsilon$. Then, if $|x - a| < \delta = \frac{1}{2}\varepsilon$, then

$$|f(x) - f(a)| = |(2x - 5) - (2a - 5)| = 2|x - a| < 2\frac{1}{2}\varepsilon = \varepsilon,$$

and so the definition of $\lim_{x \to a} f(x) = f(a)$ is satisfied.