**Question**
Find the small $\varepsilon$ expansion of the roots of $\varepsilon x^2 - 4x + 1 = 0$.

**Answer**
$\varepsilon x^2 - 4x + 1 = 0$
This is a singular perturbation as $\varepsilon$ multiplies highest power of $x$: when $\varepsilon = 0$ obtain only one root $x = \frac{1}{4}$.
CHEAT!
Exact solution is

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 - 4\varepsilon}}{2\varepsilon} \\ &= \frac{2 \pm \sqrt{4 - \varepsilon}}{\varepsilon} \\ &= \frac{2}{\varepsilon} \pm \frac{2}{\varepsilon}\left(1 - \frac{\varepsilon}{4}\right)^{\frac{1}{2}} \\ &= \frac{2}{\varepsilon} \pm \frac{2}{\varepsilon} \mp \frac{1}{4} \mp \frac{\varepsilon}{64} + O(\varepsilon^2) \\ &= \begin{cases} \frac{4}{\varepsilon} - \frac{1}{4} cdot \frac{-\varepsilon}{64} + O(\varepsilon^2) \\ \frac{1}{4} + \frac{\varepsilon}{64} + O(\varepsilon^2) \end{cases} \end{aligned}$$

so now try it the other (proper) way:
Use ansatz
$x = \frac{x_0}{\varepsilon} + x_1 + O(\varepsilon^2) \rightarrow$ for singular root $\rightarrow x_0 = 0 \Rightarrow$ regular root discord or $x_0 = 4$ actual one.
$x = x_0 + \varepsilon x_1 + O(\varepsilon^2) \rightarrow$ for regular root
Substitute into equation and balance at each order to show coefficients above.