

QUESTION

Show that when $0 < |z - 1| < 2$, then we have the Laurent expansion

$$\frac{z}{(z-1)(z-3)} = -3 \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}.$$

ANSWER

Put $w = z - 1$ and expand, as usual, about $w = 0$. We get $\frac{z}{(z-1)(z-3)} = \frac{w+1}{w(w-2)}$ and now expand about $w = 2$.