

QUESTION

Show that when $0 < |z| < 4$

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}.$$

ANSWER

$\frac{1}{4z - z^2} = \frac{1}{z(4-z)} = \frac{1}{4} \left(\frac{1}{z} + \frac{1}{4-z} \right)$ (partial fractions) $= \frac{1}{4z} + \frac{1}{4^2} \left(1 - \frac{z}{4} \right) = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$.
(This is the Laurent expansion for $0 < |z| < \infty$.)