

Question

Find an example of a function $f : \mathbf{R} \rightarrow \mathbf{R}$ with the properties

i) $f \notin R[0, 1]$

ii) $f \in L[0, 1]$

iii) $|f| \in R[0, 1]$

Is it true that if $|f| \in L[0, 1]$ then $f \in L[0, 1]$?

Answer

Example $f = \begin{cases} 1 & x \in Q \cap [0, 1] \\ -1 & x \in [0, 1] - Q \\ 0 & x \notin [0, 1] \end{cases}$

Not true, let E be a non-measurable subset of $[0, 1]$.

$$g = \begin{cases} 1 & x \in E \\ -1 & x \in [0, 1] - E \\ 0 & x \notin [0, 1] \end{cases}$$