

QUESTION

- (a) A project consists of activities  $A, B, \dots, J$  whose prerequisites are given in the table below. Draw a network, suitable for analysis by the critical path method, to represent the project. You should avoid using dummy activities, where possible.

Activity	Prerequisites	Normal duration (days)	Crash duration (days)	Cost of reduction (£)
A	-	7	5	14
B	-	6	5	10
C	-	4	3	15
D	A	5	2	12
E	B	7	4	18
F	C	9	6	30
G	B,D	8	5	27
H	E,F	7	4	45
I	G	5	2	50
J	G,H	6	5	20

Assuming that all activities have normal durations, as given in the table, write all of the earliest and latest event times on the network, and hence deduce *all* critical paths.

The table above also lists the crash duration of each activity and the corresponding cost of reducing the activity duration from its normal duration to its crash duration. It is possible to set the duration of any activity to any value between its normal and crash duration, which incurs a cost that is proportional to the reduction in duration.

Analyze how the overall project duration can be reduced by 2 days from its normal duration at minimum total cost.

- (b) A company produces a special type of flour for a bakery. The bakery orders either 1, 2 or 3 batches of flour every three months. Due to the lengthy production process, the company must produce the flour before the order arrives.

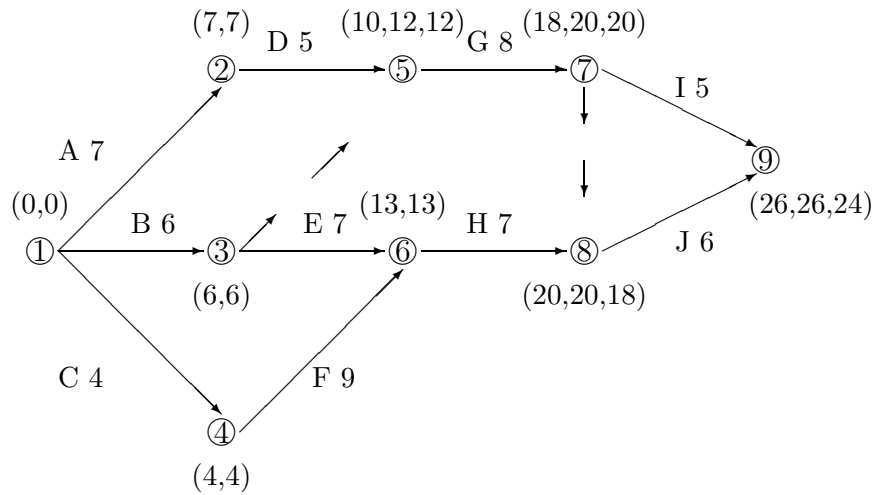
It costs £15 000 for the company to produce each batch of flour, and the bakery buys the flour at a price of \$20 000 per batch. If the bakery orders more flour than the company has produced, then the company fills the order by purchasing a substitute from another firm at a cost of \$24 000 per batch. Since the flour deteriorates over time, if the bakery orders less flour than the company has produced, then the company

reprocesses any excess batches, where each excess batch is valued at £5 000.

Based on historical data, the bakery orders 1 batch with probability 0.3, orders 2 batches with probability 0.5, and orders 3 batches with probability 0.2. Develop a decision tree to find how many batches the company should produce every three months. Also, find the maximum discount per batch that should be offered to the bakery for specifying in advance exactly how many batches it will order.

ANSWER

(a)



Critical paths are A - D - G - I, B - E - H - J, C - F - H - J

Possibilities to reduce duration are

Reduce J: unit cost 20

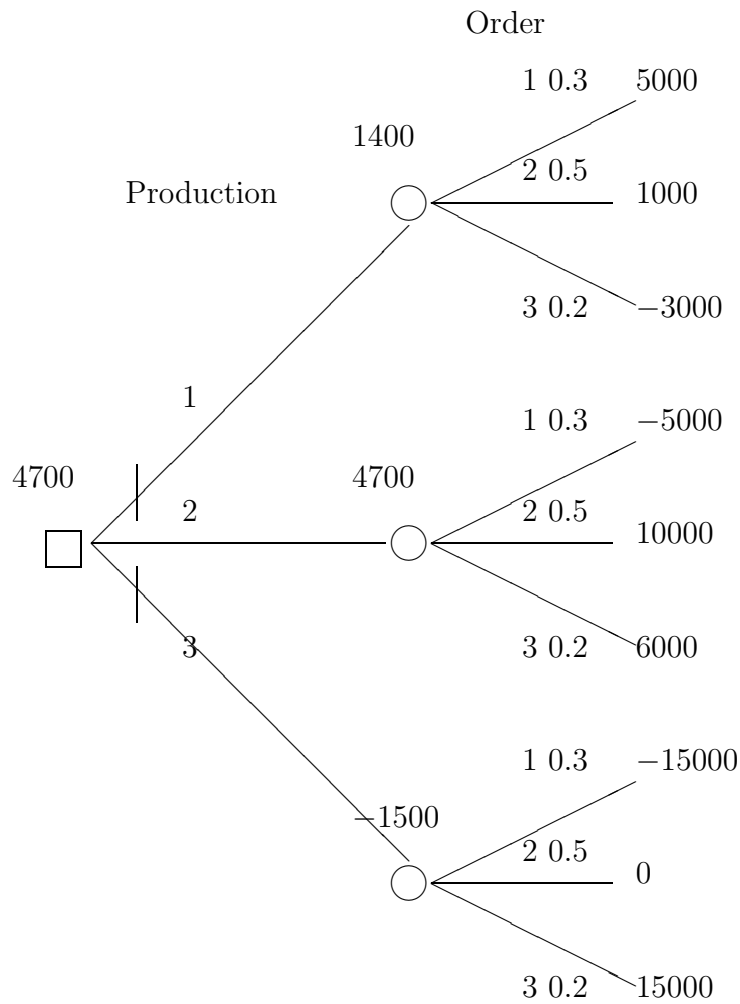
Reduce D,H: unit cost  $4+15=19$

Reduce D,E,F: unit cost  $4+6+10=20$

Reduce D,H by 2 day to 3 and 5 days.

Since the project duration becomes 24, this is the desired solution.

(b)



The optimal decision is to produce 2 batches each time.

If the bakery specifies in advance its order, then the profits are  $5000 - x$ ,  $10000 - 2x$ ,  $15000 - 3x$  with probabilities 0.3, 0.5, 0.2 where  $x$  is the discount for each batch.

The expected profit is

$$(5000 - x)0.3 + (10000 - 2x)0.5 + (15000 - 3x)0.2 = 9500 - 1.9x$$

The break even discount is given by

$$9500 - 1.9x = 4700, x = 2526.32$$