QUESTION

(a) Solve the following linear programming problem using the simplex method.

$$\begin{array}{ll} \text{Maximize} & z = 3x_1 - 10x_2 + 2x_3 + 3x_4 \\ \text{subject to} & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0 \\ & 2x_1 - 5x_2 + 4x_3 + 3x_4 = 32 \\ & 3x_1 + x_2 + 2x_3 + x_4 \geq 12 \end{array}$$

(b) An engineering company manufactures industrial machines for overseas export. Demand for the next six months, which must be satisfied, is shown in the following table.

Month	July	Aug	Sept	Oct	Nov	Dec
Demand	150	180	240	170	190	220

Capacity restrictions limit the production of machines to a maximum of 200 per month. However, it is possible to manufacture machines before they are required. In this case, there is a cost of £200 per machine in storage at the end of each month. At the start of July, there are no machines in storage, and no stock is required at the end of December.

The machines can be distributed to customers by aeroplane or by ship. Air transportation costs £1000 per machine; sea transportation costs £750 per machine. Machines to be transported by air are dispatched in the month that they are required. On the other hand, machines to be transported by ship must be dispatched one month in advance.

Write down a linear programming formulation (but do not attempt to solve it) for the problem of planning production and distribution so that demand is satisfied at minimum total cost. You may ignore any requirements for variables to be integer-valued.

ANSWER

(a) Introduce a slack variable $s \geq 0$, and artificial variables $a_1 \geq 0,\ a_2 \geq 0$

Basic	z'	z	x_1	x_2	x_3	x_4	s	a_1	a_2	
a_1	0	0	2	-5	4	3	0	1	0	32
a_2	0	0	3	1	2	1	-1	0	1	12
	1	0						1	1	0
	1	0	-5	4	-6	-4	1	0	0	-44
0	1	-3	10	-2	-3	0	0	0	0	

Basic	z'	z	x_1	x_2	x_3	x_4	s	a_1	a_2	
a_1	0	0	-4	-7	0	1	2	1	-2	8
x_3	0	0	$\frac{3}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	6
	1	0	4	7	0	-1	-2	0	3	-8
	0	1	0	11	0	-2	-1	0	1	12

Basic	z'	z	x_1	x_2	x_3	x_4	s	a_1	a_2	
s	0	0	-2	$-\frac{7}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	-1	4
x_3	0	0	$\frac{1}{2}$	$-\frac{5}{4}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	8
	1	0	0	0	0	0	0	1	1	0
0	1	-2	$\frac{15}{2}$	0	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	16	

Phase 1 ends, so discard z', a_1, a_2

Basic	z	x_1	x_2	x_3	x_4	s	
s	0	0	$-\frac{17}{2}$	4	$\frac{7}{2}$	1	36
x_1	0	1	$-\frac{5}{2}$	2	$\frac{3}{2}$	0	16
	1	0	$\frac{5}{2}$	4	$\frac{3}{2}$	0	48

Thus, we have an optimal solution

$$x_1 = 16 \; x_2 = 0 \; x_3 = 0 \; z = 48$$

(b) Let the months be labeled $1, 2, \ldots, 6$. For month t let

x_t=production

y_t=quantity sent by air

z_t= quantity sent by ship

s_t=end of month stock

$$\begin{array}{ll} \text{Minimize} & 1000(y_1 + \ldots + y_6) + 750(z_1 + \ldots + z_6) + 200(s_1 + \ldots + s_6) \\ \text{subject to} & x_t, y_t, z_t, s_t \geq 0 \; t = 1, \ldots, 6 \\ & x_t \leq 200 \; t = 1, \ldots, 6 \\ & s_{t-1} + x_t - y_t - z_t = s_t \; t = 1, \ldots, 6 \end{array}$$

$$\begin{aligned} y_1 &= 150 \\ z_1 + y_2 &= 180 \\ z_2 + y_3 &= 240 \\ z_3 + y_4 &= 170 \\ z_4 + y_5 &= 190 \\ z_5 + y_6 &= 220 \\ s_0 &= *0 \\ s_6 &= 0 \end{aligned}$$