Question
Let

$$J = \int_0^1 \frac{\sin x}{1+x^3}\,dx,$$

where the integral limits are in radians.
Carefully explaining your method, calculate J to 4 decimal places using

(i) the trapezium rule with 5 ordinates,

(ii) Simpson's rule with 5 ordinates.

Compare your answers with the exact result $J = 0.346997...$
Note: Marks will only be awarded if sufficient working is shown.

Answer

$$J = \int_0^1 \frac{\sin x}{1+x^3}\,dx$$

(i) Trapezium rule with 5 ordinates:

$J \approx \frac{d}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 + y_5)$

where $d = \frac{1-0}{5-1} = \frac{1}{4}$

$x_1 = 0 \quad x_4 = \frac{3}{4}$

$x_2 = \frac{1}{4} \quad x_5 = 1$

$x_3 = \frac{1}{2}$

$$y_i = f(x_i); \quad f(x) = \frac{\sin x}{(1+x^3)}$$

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	0	0.243598	0.426156	0.479394	0.420735

$$\begin{aligned} J &\approx \frac{1}{8}(0.420735 + 2\times(0.479394 + 0.426156 + 0.243598) + 0) \\ &= \underline{0.339879} \end{aligned}$$

NB working

$$
\begin{aligned}
&= \frac{1}{8} \times (0.420735 + 2.2983) \\
&= \frac{1}{8} \times 2.71903 \\
&= 0.339879
\end{aligned}
$$

(ii) Simpson with 5 ordinates

$J \approx \dfrac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$

4 equal segments $\Rightarrow h = \dfrac{1}{4}$ as above

and we have the <u>same</u> y_is as above.

Hence

$$
\begin{aligned}
J &\approx \frac{1}{12} \times (0 + 4 \times (0.243598 + 0.479394) \\
&\quad +2 \times (0.426156) + 0.420735) \\
&= 0.347085 \\
&= \underline{0.4407}
\end{aligned}
$$

NB working

$$
\begin{aligned}
&\frac{1}{12} \times (2.89197 + 0.852312 + 0.420735) \\
&= \frac{1}{12} \times 4.16502 \\
&= 0.347085
\end{aligned}
$$

Actual=0.346997 to get 6dp

$\Rightarrow$ Trapezium is accurate to 2% or 1dp and Simpson is accurate to 0.02% or 3dp.