Question

Given $\mathbf{u} = (4, 2, 0)$, $\mathbf{v} = (-3, 1, 1)$, $\mathbf{w} = (5, 1, 5)$, $\mathbf{s} = (1, 2, 1)$ find; (a) the angle between $\mathbf{u}$ and $\mathbf{w}$; (b) The value of μ for which for $\mathbf{u} + \mu\mathbf{v}$ is perpendicular to $\mathbf{w}$

Answer

$$\mathbf{u} = (4, 2, 0)\, \mathbf{v} = (-3, 1, 1)\, \mathbf{w} = (5, 1, 5)\, \mathbf{s} = (1, 2, 1)$$

(a) $\mathbf{u} \cdot \mathbf{w} = |\mathbf{u}||\mathbf{w}| \cos\theta$ with θ the angle between $\mathbf{u}$ and $\mathbf{w}$

$\mathbf{u} \cdot \mathbf{w} = (4, 2, 0) \cdot (5, 1, 1) = 20 + 2 + 0 = 22$

$|\mathbf{u}| = \sqrt{4^2 + 2^2 + 0^2} = \sqrt{20}$

$|\mathbf{w}| = \sqrt{5^2 + 1^2 + 1^2} = \sqrt{27}$

Hence $\cos\theta = \frac{22}{\sqrt{540}} \approx 0.946 \Rightarrow \theta \approx 18.79°$

(b) $(\mathbf{u} + \mu\mathbf{v}) \cdot \mathbf{w} = 0$ for $\mathbf{u} + \mu\mathbf{v}$ perpendicular to $\mathbf{w}$

Hence

$$
\begin{aligned}
\mathbf{u} \cdot \mathbf{w} + \mu\mathbf{v} \cdot \mathbf{w} &= 0 \\
\mu &= \frac{-\mathbf{u} \cdot \mathbf{w}}{\mathbf{v} \cdot \mathbf{w}} \\
&= \frac{-22}{(-3 \times 5 + 1 \times 1 + 1 \times 1)} \\
&= \frac{22}{13}
\end{aligned}
$$