Question
Simplify

$$\left(\frac{a+ib}{a-ib}\right)^2 - \left(\frac{a-ib}{a+ib}\right)^2$$

Answer

$$\left(\frac{a+ib}{a-ib}\right)^2 - \left(\frac{a-ib}{a+ib}\right)^2$$

$$\begin{aligned}
&= \frac{(a+ib)^4 - (a-ib)^4}{(a+ib)^2(a+ib)^2} \\
&= \frac{(a^2+2aib-b^2)^2 - (a^2-2aib-b^2)}{(a^2+b^2)^2} \\
&= \frac{a^4+4a^3ib-6a^2b^2-4aib^3+b^4-(a^4-4a^3ib-6a^2b^2+4aib^3+b^4)}{(a^2+b^2)^2} \\
&= \frac{8a^3bi-8ab^3i}{(a^2+b^2)^2} \\
&= \frac{8iab(a^2-b^2)}{(a^2+b^2)^2}
\end{aligned}$$