Question

Hint: In this question make sure you use radians for input to trigonometric functions.

Calculate to 4 decimal places of accuracy,

$$J = \int_0^1 \frac{e^x \sin x}{1 + x^2}\,dx,$$

by using,

(i) the trapezium rule with 5 ordinates;

(ii) Simpson's rule with 5 ordinates.

(iii) Compare your answers with the exact result $J = 0.608087...$, calculating the percentage error in each case.

Answer

(i) Trapezium rule with 5 ordinates:

$J \approx \frac{d}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 + y_5)$

5 ordinates $\Rightarrow$ 4 strips

Width of strips$=\frac{1-0}{4} = 0.25$

x	0.00	0.25	0.5	1.75	1.00
y	0	0.29899	0.63235	0.92354	1.14368

$$\begin{aligned}
I &= \frac{0.25}{2}((0 + 1.14368) \\
&\quad +2 \times (0.29899 + 0.63235 + 0.92354)) \\
&= 0.125(1.14368 + 3.70976) \\
&= 0.125 \times 4.85344 \\
&= 0.60668... \\
&= 0.6067...
\end{aligned}$$

(ii) Simpson with 5 ordinates

$I \approx \frac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$

Ordinates y_i are the same as above (h too).

Hence

$$
\begin{aligned}
I &\approx \frac{0.25}{2}((0 + 1.14368) + 4 \times (0.29899 + 0.92354) \\
&\quad +2 \times 0.63235) \\
&\approx \frac{0.25}{3}(1.14368 + 4.89012 + 1.26470) \\
&= \frac{0.25}{3}[7.29850] \\
&= 0.6082
\end{aligned}
$$

(iii) Trap is accurate to $\left|\dfrac{(0.608087 - 0.6066)}{0.608087} \times 100\right| = 0.24\%$

Simpson is accurate to $\left|\dfrac{(0.608087 - 0.60821)}{0.608087} \times 100\right| = 0.02\%$

Simpson is 10 times better!!