

**Question**

Let  $f_a(x) = x^4 - 3ax^2 + x + 2a^2$ . Show that fixed points for  $f_a$  are created as  $a$  increases through zero, but this is not a saddle-node bifurcation. Which conditions in Theorem A fail?

**Answer**

$f_a(x) = x$  where  $x^4 - 3ax^2 + 2a^2 = 0$  i.e.  $(x^2 - a)(x^2 - 2a) = 0$ . No solutions for  $a > 0$ ; solutions  $\pm a, \pm\sqrt{2}a$  for  $a > 0$ . We have  $f'_0(0) = 1$  (OK for saddle-node) but  $\left. \frac{\partial}{\partial a} f_a(0) \right|_{a=0} = 4a|_{a=0} = 0$ : second condition fails.