QUESTION

Find an integer n that can be written as a sum of two squares in two essentially different ways; i.e.$n = u^2 + v^2 = w^2 + x^2$ where $w \neq \pm u, \pm v$.

[Hint: recall $(a^2+b^2)(c^2+d^2) = (ac+bd)^2 + (ad-bc)^2$. What happens when we exchange c and d?]

ANSWER

$(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2$, and $(a^2 + b^2)(d^2 + c^2) = (ad + bc)^2 + (ac - bd)^2$.

These two expressions could turn out to be different, e.g. $5 = 2^2 + 1^2$, and $17 = 4^2 + 1^2$. Thus, as above, we have $85 = 5.17 = (2^2 + 1^2)(4^2 + 1^2) = (8 + 1)^2 + (2 - 4)^2 = 9^2 + 2^2$, and $85 = 5.17 = (2^2 + 1^2)(1^2 + 4^2) = (2 + 4)^2)(8 - 1)^2 = 6^2 + 7^2$.

[Plenty of other examples are available- so your answer may well be different.]