QUESTION

Represent 765 as a sum of two squares. Represent 323 as a sum of four squares.

[Hint: Factorise them first]

ANSWER

$765 = 5.153 = 5.3.51 = 5.3^2.17$.

Now $5 = 2^2 + 1^2$, so $5.3^2 = (2^2 + 1^2).3^2 = (2.3)^2 + (1.3)^2 = 6^2 + 3^2$. Also $17 = 4^2 + 1^2$. Hence $5.3^2.17 = (6^2 + 3^2)(4^2 + 1^2)$, and using the identity $(a^2+b^2)(c^2+d^2) = (ac+bd)^2+(ad-bc)^2$, we get $5.3^2.17 = (24+3)^2+(6-12)^2 = 27^2 + 6^2$.

$323 = 17.19$

Now $17 = 4^2 + 1^2 = 4^2 + 1^2 + 0^2 + 0^2$, and $19 = 4^2 + 1^2 + 1^2 + 1^2$, so using the identity

$$
\begin{aligned}
(a^2 + b^2 + c^2 + d^2)(e^2 + f^2 + g^2 + h^2) &= (ae + bf + cg + dh)^2 \\
&+ (af - be + ch - dg)^2 \\
&+ (ag - bh - ce + df)^2 \\
&+ (ah + bg - cf - de)^2
\end{aligned}
$$

we get $17.19 = (16+1+0+0)^2+(4-4+0-0)^2+(4-1-0+0)^2+(4+1-0-0)^2 = 17^2 + 0^2 + 3^2 + 5^2$