QUESTION

Show that the equation $x^2 \equiv 1 \bmod 8$ has more than 2 incongruent solutions.
Why does this not contradict Lagrange's theorem?

ANSWER

Each of the residues ± 1, ± 3 satisfies $x^2 \equiv 1 \bmod 8$, so this equation has more than 2 incongruent solutions. This does not contradict Lagrange's theorem, as the theorem refers to prime moduli only, and 8 is not prime.